EE341 Fall 2013
Problem Set 2 Solutions

1)

3.5.3) From Fourier Transform table 3) and time shifting property assuming \( \beta > 0 \) we have that
\[ h(t) = \exp(-\beta |t - t_0|) \]
The impulse response has infinite support and is therefore noncausal. However, if \( t_0 > 0 \) is large the values for \( h(t) < 0 \) are small. We can choose \( t_0 \) such that 99% of the energy occurs for positive values of \( h(t) \). Then for this value of \( t_0 \) we can approximate \( h(t) \) by
\[ g(t) = h(t)u(t) \]
Because of symmetry this occurs when
\[ \int_{-\infty}^{\infty} \exp(-2\beta t)dt = .02/(2\beta) \]
or \( t_0 = -1/(2\beta) \log(.02) \). We can also compute frequency response which is given by
\[ G(f) = H(f) - \int_{-\infty}^{0} \exp(\beta(t - t_0)) \exp(-j2\pi ft)dt = H(f) - \exp(-\beta t_0)/(\beta - j2\pi f) \]

3.6.2) First note that the impulse response of channel with multipath is given by
\[ h(t) = \delta(t - t_d) + \alpha \delta(t - t_d - \Delta t) \]
and the frequency response is \( H(f) = \exp(-j2\pi f t_d)/(1 + \alpha \exp(-j2\pi f \Delta t)) \). Then if we let \( H_{eq}(f) = 1/(1 + \alpha \exp(-j2\pi f \Delta t)) \), then channel will be equalized with no distortion as the overall response is just a delay of \( t_d \). For \( \alpha \) small this can be approximated by a finite tap delay line where the delays are \( \Delta t \) with \( a_i = (-\alpha)^i \) where \( 0 \leq i \leq n \).

2) Matlab plots of signal and Hilbert transform are shown on first page of plots.

a) \( x(t) = \sin^2(10\pi t) \cos(5\pi t) = .5 \cos(5\pi t) - .25 \cos(15\pi t) - .25 \cos(25\pi t) \). \( \hat{x}(t) = .5 \sin(5\pi t) - .25 \sin(15\pi t) - .25 \sin(25\pi t) \).

b) \( y(t) = \frac{2t}{1+t^2} \), use partial fraction expansion to get \( Y(f) = 2\pi(-j \exp(-2\pi a f)u(f) + j \exp(2\pi a f)u(-f)) \), \( \hat{Y}(f) = -2\pi \exp(-2\pi a |f|) \). Taking inverse Fourier Transform we get that \( \hat{y}(t) = \frac{-2a}{1+t^2} \). Note that matlab simulations are not accurate for part b) as we need to sample points with \(-t\) with larger magnitude inputs.

c) \( z(t) = sinc(t) \cos(100\pi t) \)

To find the Hilbert transform in matlab use command \( xhat = \text{imag(hilbert(x))} \); Of the three different signals only \( z(t) \) passes through Hilbert transform relatively undistorted.

3) Matlab plots are shown on the last three pages.

a) First two plots are of signal \( x(t) \) and its Fourier Transform \( X \).

b) Next two plots and first three plots of following page are of outputs of RC filter. When \( RC \leq .03 \) \( y(t) \) is a relatively undistorted version of \( x(t) \).
c) First two plots of last page are of magnitude and phase of given filter. The rest of the plots are the Fourier Transform of signals p, q, r, s. Note that the distortionless bands are from 1KHz to 4KHz. Signals p and r get through undistorted. q has amplitude distortion and s has phase distortion.

4) Distortionless bands are from 500 to 1000 Hz. x(t) has phase distortion, y(t) undistorted, and z(t) has amplitude and phase distortion.