EE213 Exam 1 Solutions

1)

a) The circuit is a voltage divider and we have $V_{Th} = V_{in}(10/j)/((10/j) + 10) = 7.0711\angle(-45^o)$. The Thevenin impedence is given by $Z_{Th} = 10||(10/j) + 10 + 10j = (15 + 5j)\Omega$.

b) $Z_L = Z_{Th}^* = (15 - 5j)\Omega$ and maximal power is $P = |V_{Th}|^2/(8 * Re(Z_{Th})) = 5/12 = .4167W$.

2)

a) Let the voltage at the input of the opamp be labeled as B, then $B = V_{out}/2$.

b) Let the voltage at the node between R_1 and the two capacitors be A, then we have at input of negative input of opamp

$$V_{out}/2(sC_2 + 1/R_2) - AsC_2 - V_{out}/R_2 = 0$$

Manipulating equation we have $A = V_{out}(sC_2 - 1/R_2)/(2sC_2)$. At other node we have

$$A(1/R_1 + sC_1 + sC_2) - V_{out}(sC_1 + sC_2/2) = V_{in}/R_1$$

c) Manipulating first equation we have $A = V_{out}(sC_2 - 1/R_2)/(2sC_2)$. Then substituting into second equation we have that

$$V_{out}((sC_2) - 1/R_2)(1/R_1 + sC_1 + sC_2)/(2sC_2) - sC_1 - sC_2/2) = V_{in}/R_1.$$

Then we have that

$$H(s) = \frac{-2s/(R_1C_1)}{s^2 + s(1/(R_2C_1) + 1/(R_2C_2) - 1/(R_1C_1)) + 1/(R_1C_1R_2C_2)}$$

3) Let B be the voltage at the node to the right of R_1 and D the voltage at the output of the left opamp.

a)
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>> syms R R1 R2 R3 R4 C C2  Vin Vout B D s H t vin vout;
>> mat=[s*(C+C2)+1/R1 -s*C2 0; -s*C -1/R2 0; 0 -1/R3 -1/R]; vec=[1/R1;0; 1/R4];
>> H =simplify([0 0 1]*inv(mat)*vec)
                    H = (C*R*R2*s)/(R3*(C*R1*s + C2*R1*s + C*C2*R1*R2*s^2 + 1)) - R/R4
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b)
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>>  Vin = laplace(vin); Vout = Vin * H;  vout = ilaplace(Vout)
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c) At both high frequencies capacitors short and at low frequencies capacitors open. For both cases we have that the right opamp is an inverting amplifier with $V_{out} = -R/R_4V_{in}$.

4)

$$H(s) = \frac{2s+3}{s+4} = 2 - \frac{5}{s+4}$$

a) $h(t) = 2\delta(t) - 5\exp(-4t)u(t)$.

b) $s(t) = \mathcal{L}^{-1}((2s+3)/s(s+4)) = (.75 + 1.25\exp(-4t))u(t)$.

c) $y(t) = 10|H(j2)|\cos(2t + \pi/4 + \angle(H(j2)) = 11.18\cos(2t - 18.4349^o)$.

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1)

a) The circuit is a voltage divider and we have $V_{Th} = V_{in}(20j)/(10+20j) = 17.8885\angle(26.5651^o)$. The Thevenin impedence is given by $Z_{Th} = 10||(20j) + 30 - 10j = (38 - 6jj)\Omega$.

b) $Z_L = Z_{Th}^* = (38+6j)\Omega$ and maximal power is $P = |V_{Th}|^2/(8 * Re(Z_{Th})) = 320/304 = 1.0526W$.

2)

a) Let the voltage at the input of the opamp be labeled as B, then $B = V_{out}/2$.

b) Let the voltage at the node between R_1 and the two capacitors be A, then we have at input of negative input of opamp

$$V_{out}/2(sC_2 + 1/R_2) - A/R_2 = 0$$

Manipulating equation we have $A = V_{out}(sC_2 + 1/R_2)/(2/R_2)$. At other node we have

$$A(1/R_1 + sC_1 + 1/R_2) - V_{out}(sC_1 + 1/(2R_2)) = V_{in}/R_1$$

c) Manipulating first equation we have $A = V_{out}(sC_2 + 1/R_2)/(1/2R_2)$. Then substituting into second equation we have that

$$V_{out}((sR_2C_2) + 1)(1/R_1 + sC_1 + 1/R_2)/2 - sC_1 - 1/(2R_2)) = V_{in}/R_1.$$

Then we have that

$$H(s) = \frac{2/(R_1C_1R_2C_2)}{s^2 + s(1/(R_1C_1) + 1/(R_2C_1) - 1/(R_2C_2)) + 1/(R_1C_1R_2C_2)}$$

3) Let B be the voltage at the output of the leftmost opamp and D be the voltage at the output of the center opamp.

a)
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>> syms R R1 R2 R3 R4 R6 C1 C2  Vin Vout B D s H t vin vout;
>> mat=[s*C1+1/R2 0  1/R3; 1/R4  s*C2 0; 0 1/R  1/R6]; vec=[-1/R1;0; 0];
>> H =simplify([0 0 1]*inv(mat)*vec)
                    H = (R2*R3*R6)/(R1*(C1*C2*R*R2*R3*R4*s^2 + C2*R*R3*R4*s + R2*R6))
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b)
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>>  Vin = laplace(vin); Vout = Vin * H;  vout = ilaplace(Vout)
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c) At high frequencies the capacitors short and $H(s) = 0$. At low frequencies the capacitors open and the circuit acts as an inverting amplifier with $H(s) = -R_3/R_1$.

4)

$$H(s) = \frac{s+3}{s+2} = 1 + \frac{1}{s+2}$$

a) $h(t) = \delta(t) + \exp(-2t)u(t)$.

b) $s(t) = \mathcal{L}^{-1}((2s+3)/s(s+4)) = (.1.5 - 0.5\exp(-2t))u(t)$.

c) $y(t) = 10|H(j2)|\cos(2t + \pi/4 + \angle(H(j2)) = 12.748\cos(2t + 33.6901^o)$.