EE213 Exam 2 Solutions

1)

a) Perform convolution $y(t) = x(t) * h(t)$. Here we will flip $h(t)$. This results in

$$y(t) = \begin{cases} 0 & t < 0 \\ \int_0^t \exp(-2\tau)\exp(-2(t-\tau))d\tau = t\exp(-2t) & 0 \le t \le 3 \\ \int_0^3 \exp(-2\tau)\exp(-2(t-\tau))d\tau = 3\exp(-2t) & t > 3 \end{cases}$$

b) First find step response $s(t) = \int_0^t \exp(-2\tau)d\tau = 1/2(1-\exp(-2t))u(t)$. Then $n(t) = s(\infty) - s(t) = 1/2 - s(t)$. Therefore

$$n(t) = \begin{cases} 1/2 & t < 0 \\ 1/2\exp(-2t) & t \ge 0 \end{cases}$$

2) This problem can be solved by either using a state space approach or by adding step voltage sources for each initial condition. Here we use the state space approach. The state is given by $x(t) = [v_{C_1}(t), v_{C_2}(t)]^T$ where both voltages are from left to right. Note that $out(t) = -v_{C_2}(t)$. We then have

$$C_1 \frac{dv_{C_1}(t)}{dt} = -v_{C_1}(t)(1/R_1 + 1/R_2) + v_{C_2}(t)/R_1, \quad C_2 \frac{dv_{C_2}(t)}{dt} = -v_{C_1}(t)/R_1$$

Substituting values we have that

$$A = \begin{bmatrix} -3 & 1 \\ -2 & 0 \end{bmatrix}, \; b = [0,0]^T, \; c = [0,-1], d = 0$$

We then get that $Out(s) = c(sI - A)^{-1}x(0) = [0 - 1]\begin{bmatrix} s+3 & -1 \\ 2 & s \end{bmatrix}^{-1}[2,1]^T = \frac{1-s}{s^2+3s+2}$. Taking the inverse Laplace Transform we get that

$$out(t) = (-3\exp(-2t) + 2\exp(-t))u(t)$$

3)

a) Let all currents and voltages go from left to right or top to bottom. Then define $x(t) = [v_C(t), i_{L_1}(t), i_{L_2}(t)]^T$.

b) Write node and mesh equations to get that

$$C\frac{dv_C(t)}{dt} = -v_C(t)/R_2 + i_{L_2}(t)$$

$$L_1 \frac{d_{L_1}(t)}{dt} = -i_{L_1}(t)R_1 - i_{L_2}(t)R_1 + in(t)$$

$$L_2 \frac{d_{L_2}(t)}{dt} = -v_C(t) - i_{L_1}(t)R_1 - i_{L_2}(t)R_1 + in(t)$$

$$out(t) = v_C(t)$$

Substituting values we have that

$$A = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & -1 \\ -1 & -2 & -2 \end{bmatrix},\ b = [0,1,1]^T,\ c = [1,0,0], d = 0$$

c) .

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>> syms s H Inp Out t inp out;
>>A=[-1 0 1;0 -1 -1; -1 -2 -2];B=[0; 1; 1];C=[1 0 0];D=0;
>> H=D+C*inv(s*eye(size(A))-A)*B;
>> inp = exp(-2*t)*heaviside(t); Inp=laplace(inp);
>> Out = H*Inp + C*inv(s*eye(size(A))-A)*[1 1 1]';
>> out = ilaplace(Out) *heaviside(t);
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4) $H_1(s)$ has a zero at 0 and poles at $-2 \pm j2$. Step response matches Waimea.
$H_2(s)$ has a double zeroes at 0 and double poles at -2. Step response matches Kalalau.
$H_3(s)$ has poles at $-2 \pm j2$. Step response matches Mahanaloha.
$H_4(s)$ has double poles at -2. Step response matches Honopu.

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1)

a) Perform convolution $y(t) = x(t) * h(t)$. Here we will flip $h(t)$. This results in

$$y(t) = \begin{cases} 0 & t < 0 \\ \int_0^t \exp(\tau)\exp(-(t-\tau))d\tau = 1/2(\exp(t) - \exp(-t)) & 0 \le t \le 2 \\ \int_0^2 \exp(\tau)\exp(-(t-\tau))d\tau = 1/2(\exp(4-t) - \exp(-t)) & t > 2 \end{cases}$$

b) First find step response $s(t) = \int_0^t \exp(-\tau)d\tau = (1 - \exp(-t))u(t)$. Then $n(t) = s(\infty) - s(t) = 1 - s(t)$. Therefore

$$n(t) = \begin{cases} 1 & t < 0 \\ \exp(-t) & t \ge 0 \end{cases}$$

2) This problem can be solved by either using a state space approach or by adding step voltage sources for each initial condition. Here we use the state space approach. The state is given by $x(t) = [v_{C_1}(t), v_{C_2}(t)]^T$ where both voltages are from left to right. Note that $out(t) = -v_{C_2}(t)$. We then have

$$C_1 \frac{dv_{C_1}(t)}{dt} = -v_{C_1}(t)(1/R_1 + 1/R_2) + v_{C_2}(t)/R_1, \quad C_2 \frac{dv_{C_2}(t)}{dt} = -v_{C_1}(t)/R_1$$

Substituting values we have that

$$A = \begin{bmatrix} -2 & 1 \\ -1 & 0 \end{bmatrix},\ b = [0,0]^T,\ c = [0,-1], d = 0$$

We then get that $Out(s) = c(sI - A)^{-1}x(0) = [0\ -1] \begin{bmatrix} s+2 & -1 \\ 1 & s \end{bmatrix}^{-1} [2,1]^T = \frac{-2s-3}{s^2+2s+1}$. Taking the inverse Laplace Transform we get that

$$out(t) = -(2t\exp(-t) + \exp(-t))u(t)$$

3)

a) Let all currents and voltages go from left to right or top to bottom. Then define $x(t) = [v_C(t), i_{L_1}(t), i_{L_2}(t)]^T$.

b) Write node and mesh equations to get that

$$C\frac{dv_C(t)}{dt} = -v_C(t)/R_1 - i_{L_1}(t) + in(t)/R_1$$

$$L_1\frac{d_{L_1}(t)}{dt} = v_C(t) - i_{L_1}(t)R_2 + i_{L_2}(t)R_2$$

$$L_2\frac{d_{L_2}(t)}{dt} = i_{L_1}(t)R_2 - i_{L_2}(t)R_2$$

$$out(t) = i_{L_1}(t)R_2 - i_{L_2}(t)R_2$$

Substituting values we have that

$$A = \begin{bmatrix} -1 & -1 & 0 \\ 1 & -1 & 1 \\ 1 & 2 & -2 \end{bmatrix},\ b = [1,0,0]^T,\ c = [0,1,-1], d = 0$$

c) .

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>> syms s H Inp Out t inp out;
>>A=[-1 -1 0;1 -1 1; 0 2 -2];B=[1; 0; 0];C=[0 1 -1];D=0;
>> H=D+C*inv(s*eye(size(A))-A)*B;
>> inp = exp(-4*t)*heaviside(t); Inp=laplace(inp);
>> Out = H*Inp + C*inv(s*eye(size(A))-A)*[1 1 1]';
>> out = ilaplace(Out) *heaviside(t);
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4) $H_1(s)$ has poles at -1, -5. Step response matches Honopu.
$H_2(s)$ has double zeroes at 0 and poles at $-3 \pm j3$. Step response matches Mahanaloha.
$H_3(s)$ has a zero at 0 and poles at -1, -5. Step response matches Waimea.
$H_4(s)$ has a zero at 0 and poles at $-3 \pm j3$. Step response matches Kalalau.