EE213 Spring 1999
Exam 2 Solutions

1) Add a four volt voltage source in series with the capacitor and a two amp current source in parallel with the inductor. Using impedance analysis we get a node equation at the positive input to the opamp.

\[ U(s)4s - (3/s)4s + W(s) + W(s)/(3s) = 2/s \]

Rearrange terms and with a step input \( U(s) = 1/s \) we get \( W(s) = (12s + 2)/(s + 1/3) \) yielding

\[ w(t) = 12\delta(t) - 2e^{-t/3}u(t). \]

2) First note that \( H(s) = 2/(s + 2) \) and \( h(t) = 2e^{-2t}u(t) \). The convolution has two nonzero cases.

\[ y(t) = \begin{cases} 2(t + 1)e^{-2t}, & \text{if } -1 \leq t \leq 0 \\ 2e^{-2t}, & \text{if } 0 \leq t \\ 0, & \text{otherwise.} \end{cases} \]

matlab code:
```
dt=.01;t=-1:dt:9;x=[exp(-2*pi*(1:100)) zeros(1,901)];
h = [ zeros(1,100) 2*pi exp(-2*pi*(1:101:1001)); y = conv(x,h)*dt; plot(t,y(1:1001));
```

3) In state space form we have that

\[ A = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad c = [3 \ 0], \quad d = 0; \]

Transfer function is then \( H(s) = 3/(s - 1) \). First we find ZSR where \( ZSR(s) = 3/[(s - 1)(s - 3)] \) and \( ZSR(t) = (-1.5e^t + 1.5e^{3t})u(t) \). We then find ZIR and get that \( ZIR(t) = 6e^t u(t) \). Finally, the total solution is \( y(t) = (4.5e^t + 1.5e^{3t})u(t) \).

4) Let \( x = [v \ i]' \), we then get that

\[ A = \begin{bmatrix} 0 & 1/C \\ -1/L & -R/L \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ 1/L \end{bmatrix}, \quad c = [-2 \ -2R], \quad d = 2; \]

Using matlab commands:
```
syms s L R C
H = 2-2*[ -1 \ -R]*inv(s*eye(3) -[0 1/C; -1/L, -R/L])*[ 0;1/L];
```

mean=24.5, std=8.5.