1. (16 points) The impulse response $h(t)$ of a circuit and the input $v_i(t)$ to the circuit are given below. In this problem, you are asked to demonstrate that you know how to use the convolution integral to find $v_0(t)$.

(i) Specify each of the different time intervals for the different functional forms for $v_0(t)$.
(ii) For each interval, express $v_0(t)$ as a definite integral. Please use the following form of the convolution integral

$$v_0(t) = \int_0^t h(\lambda)v_i(t - \lambda)d\lambda.$$ 

Full credit will be given for the correct definite integrals and no additional credit will be given for its evaluation by calculus.

$h(t)$ is the function $t$ for $0 \leq t \leq 10$, $20 - t$ for $1- \leq t \leq 20$ and 0 otherwise. $v_i(t)$ is $10t$ for $0 \leq t \leq 1$ and 0 otherwise.

2. (16 points) This is a continuation of problem 1.

(a) Express $h(t)$ and $v_i(t)$ in terms of delayed step functions.
(b) Determine the Laplace transforms of $h(t)$ and $v_i(t)$.
(c) Determine $V_0(s)$.
(d) Invert $V_0(s)$ to determine $v_0(t)$.

3. (15 points) A second order Butterworth high-pass filter circuit is given below. In this problem, you are asked to do the initial work to derive the transfer function $H(s)$ in terms of $R_1$, $R_2$, and $C$.

(a) Write the nodal equations for $A$ and the noninverting input. The solution to these two simultaneous equations result in an expression for $V_0(s)$ as a function of $V_i(s)$. Then $H(s)$ is easily determined.
(b) For $C = 1$, determine $R_1$ and $R_2$ to synthesize the transfer function

$$H(s) = \frac{s^2}{s^2 + 10s + 1}.$$ 

Insert Figure 16.25

4. (22 points) The circuit below is a third order Butterworth low-pass filter with a corner frequency of 1 rad/sec. The input voltage $v_g(t)$ to the circuit is the periodic triangular waveform given below. It desired to synthesize a circuit with the transfer function

$$H(s) = \frac{V_0(s)}{V_g(s)} = \frac{1}{s^3 + 2s^2 + 2s + 1}.$$ 

(a) For $R = 1$, specify the capacitor values.
(b) Express $v_g(t)$ as a Fourier series.
(c) Approximate $v_0(t)$ by expressing $v_0(t)$ by its Fourier series including harmonics up to the fifth one.
(d) Suppose part (c) was repeated for a third order Butterworth high-pass filter. Which approximation would be better? Explain.

*Insert Figure P17.50*

5. (18 points) In the circuit below, assume that there is no energy stored at \( t = 0 \).
(a) Let \( V_0^1(s) \) be the Laplace transform of the output due to only to the voltage source. Find \( V_0^1(s) \).
(b) Using superposition, determine \( V_0(s) \).

*Insert Figure P14.47 where (i) the capacitor and inductor are interchanged, (ii) the resistances values are changed to 100 and 10, (iii) the capacitance is 1F and (iv) the inductance is 1H.*

6. (13 points)
(a) Suppose

\[
V_0(s) = \frac{5(s + 2)^2}{s(s + 1)^3}.
\]

Determine \( v_0(t) \).
(b) Suppose the Laplace transform of \( f(t)u(t) \) is \( F(s) \). Prove that the Laplace transform of

\[
\int_0^t f(x) \, dx
\]

is \( \frac{F(s)}{s} \).