1) Let \( w(t) \) be the voltage at the positive input to the opamp and \( z(t) \) be the voltage between the two capacitors and \( R_1 \). First note that

\[
W = OUT \frac{\gamma}{\gamma + 1}
\]

and

\[
Z = OUT \left( \frac{\gamma}{\gamma + 1} \frac{1}{R_3} - 1/(\gamma + 1) + \frac{\gamma}{\gamma + 1} s C_2 \right).
\]

We then write the following node equation at \( Z \).

\[
Z(1/R_1 + s(C_1 + C_2)) - s C_1 OUT - s C_2 W = IN/R_1
\]

Then substitute for \( W \) and \( Z \) and rearrange to get that

\[
H(s) = \frac{OUT(s)/IN(s)}{s^2 + ((C_1 + C_2)(1-\gamma R_2/R_3)/(R_2 C_1 C_2) - \gamma/(R_1 C_1)s + (1-\gamma R_2/R_3)/(R_1 R_2 C_1 C_2)} = \frac{-s(\gamma + 1)/(R_1 C_1)}{s^2 + ((C_1 + C_2)(1-\gamma R_2/R_3)/(R_2 C_1 C_2) - \gamma/(R_1 C_1)s + (1-\gamma R_2/R_3)/(R_1 R_2 C_1 C_2)}
\]

This is a bandpass biquad filter. Let \( \omega_0^2 = (1-\gamma R_2/R_3)/(R_1 R_2 C_1 C_2) \) and then all other substitutions follow.

b) Note that we can get rid of \( R_3 \) and then circuit is basically the same as that in Homework 5. Let \( R_3 = \infty, C_1 = C_2 = 1/\omega_0 \). This requires that \( R_1 R_2 = 1 \). We then have two equations and two unknowns.

\[
g = \gamma R_2
\]

\[
b = 2/R_2 - \gamma R_2
\]

solving we get that \( R_1 = .5(\beta + g), R_2 = 2/(\beta + g) \), and \( \gamma = .5g(b + g) \).

2)

a) Note that it is easy to find relationship between \( V_2 \) and \( V_1 \). We have an integrator and inverter in cascade to get

\[
V_2 = V_1 R_6/(R_2 R_3 s C_2).
\]

We can then write a node equation at positive input to first opamp to get that

\[
IN/R_4 = -V_1 (1/R_1 + s C_1 + R_6/(R_2 R_3 s C_2)).
\]

The transfer function with appropriately set parameters follows. Here we have a bandpass filter.
b) Again set \( C_1 = C_2 = \omega_0 \), \( R_2 = R_3 = R_5 = R_6 = 1 \). We then have that \( R_1 = 1/\beta \) and \( R_4 = 1/g \).

c) See a) with correction that numerator of transfer function is \(-g_2\) with \( g_2 = R_5/R_4 \). Here we have a lowpass filter.

d) Same as c) except \( R_4 = 1/g_2 \).

3)

a) Magnitude and phase are both monotone decreasing. For order \( n \) filter: low frequency magnitude (below cutoff freq.) is essentially flat (flatter for higher \( n \)), high frequency magnitude (above cutoff freq.) decreases at 20ndB/decade (with -3dB value at cutoff freq.), low frequency phase is essentially flat at 0 deg (flatter for higher \( n \), and high frequency phase is \(-90n\) deg (with \(-45n\) deg value at cutoff freq.). Cutoff frequency scales frequency.

b) For order \( n \) filter: \( n \) poles no zeros. Poles are equally spaced on circle in LHP with radius \( w_n \). When \( n \) is odd one pole on real axis with all other poles in complex conjugate pairs.

c) Magnitude monotone increasing and phase monotone decreasing. For order \( n \) filter: high frequency magnitude (above cutoff freq.) is essentially flat (flatter for higher \( n \)), low frequency magnitude (below cutoff freq.) increases at 20ndB/decade (with -3dB value at cutoff freq.), low frequency phase is essentially flat at 90n deg (flatter for higher \( n \), and high frequency phase is flat at 0 deg (with \(-45n\) deg value at cutoff freq.). Cutoff frequency scales frequency.

d) Pole structure same as LP filter. Also have \( n \) zeros at origin.

4)

a) Phase is monotone decreasing and magnitude has ripples in passband. \( R_p \) determines magnitude of ripples. Larger \( R_p \) result in faster dropoff in magnitude and phase at higher frequencies (around cutoff). filter: low frequency magnitude (below cutoff freq.) has equiripples (\( n \) critical points), high frequency magnitude (above cutoff freq) decreases at 20ndB/decade (with -3dB value at cutoff freq.), low frequency phase is essentially flat at 0 deg (flatter for higher \( n \), and high frequency phase is \(-90n\) deg. Cutoff frequency scales frequency.

b) For order \( n \) filter: \( n \) poles no zeros. Poles are in LHP on an ellipse with dominant axis along imaginary axis. When \( n \) is odd one pole on real axis with all other poles in complex conjugate pairs.
c) Phase monotone decreasing and magnitude has ripples in passband. $R_p$ determines magnitude of ripples. Larger $R_p$ result in faster dropoff in magnitude and phase at higher frequencies (around cutoff). For order $n$ filter: high frequency magnitude (above cutoff freq.) has equiripples ($n$ critical points), low frequency magnitude (below cutoff freq.) increases at 20dB/decade (with -3dB value at cutoff freq.), low frequency phase is essentially flat at 90deg (flatter for higher $n$, and high frequency phase is flat at 0deg. Cutoff frequency scales frequency.

d) For order $n$ filter: $n$ poles and $n$ zeros at origin. Poles are in LHP on an ellipse with dominant axis along real axis. When $n$ is odd one pole on real axis with all other poles in complex conjugate pairs.

e) See 3b) and 4b). Chebyshev filter poles on ellipse and much closer to imaginary axis than Butterworth filter poles that lie on circle. Butterworth filter poles all have same magnitude, but Chebyshev filter poles vary in magnitude.

Figure 1: Magnitude and Phase of Butterworth and Chebyshev1 Filters