a) Refer to Fig. P8.30. We first find the initial conditions of the circuit by considering the operation of the circuit when the switch is open for a long time. The capacitor is opened. We then have that $v_C(0) = 20V$ and $i_L(0) = 0A$. To handle initial conditions we add a current source in parallel with the capacitor whose Laplace transform is $I_C(s) = 20/(1/Cs) = 20C$ where $C = 5\mu F$. We then apply a source transformation to get an external current source $I_i(s) = 160m/s$ in parallel with $I_C(s)$. These two current sources are added to get $I(s) = 20C + 160m/s$ and are in parallel with two resistors, a capacitor, and an inductor. Equivalent resistor is $25\Omega$ and inductor is $312.5mH$. This is just a current divider circuit and we get that the transfer function is

$$H(s) = \frac{I_L(s)}{I(s)} = \frac{1/sL}{sC + 1/R + 1/sL} = \frac{\omega_0^2}{s^2 + 2s\omega_0/Q + \omega_0^2}$$

(1)

Here we have $\omega_0 = 800$ and $Q = 1$. This is a critically damped system with repeated poles at $s = -800$. We then get that

$$I_L(s) = \frac{1m(s = 2\omega_0)\omega_0^2}{s(s + \omega_0)^2} = .1m(2/s - 2/(s + \omega_0) - \omega_0/(s + \omega_0)^2).$$

By taking the inverse Laplace transform we get that

$$i_L(t) = (160 - 160e^{-800t} - 64000e^{-800t})u(t)mA.$$

b) Refer to Fig. P8.32. Again we find the initial conditions by opening the capacitor and shorting the inductors to get $v_C(0) = 50V$ and $i_L(0) = 100mA$. We then apply a source transformation to get $I_i(s) = 20m/s$. the circuit is again a parallel RLC circuit with $R = 2500\Omega$, $C = .25\mu F$, and $L = 4H$. The transfer function is the same as equation (1) with $\omega_0 = 1000$ and $1/Q = 1/(.64)$. This is an underdamped system. The initial conditions are handled the same was as part a) with $I_L(s) = -100mA$ and $I_C(s) = 50C$. We add all the current sources to get $I(s) = -80m/s + .0125mA$. The output current is therefore

$$I_L(s) = I(s) \ast H(s) + 100m/s.$$

The last term accounts for the $100mA$ current source. By inverting we get

$$i_L(t) = (20 + 80e^{-800t}\cos(600t) + 127.5e^{-800t}\sin(600t))u(t)mA.$$
2) Refer to HW #4 problem 2) and let \( C_1 = C_2 \). Let \( v_i(t) = 1V \) and assume that one volt has been applied to the circuit for a long time. Then after this long period of time (at time 0), the voltage source \( v_i(t) \) is turned off. We open the capacitors and note that at time 0, \( v_o(0) = 0 \) and \( x(0) = 1V \) where \( x \) is the voltage at node between two capacitors. We therefore have that voltage at time 0 across each capacitor is 1V.

Add two 1V voltage sources across each capacitor. We then write two node equations at \( x \) and at input of negative terminal of opamp to get

\[
(2sC + 1/R_1)X - sCV_o = 2C
\]

and

\[
sC X + (1/R_2)V_o = C.
\]

These equations can be solved manually or by matlab to get

\[
V_o(s) = \frac{\omega_0 Q}{s^2 + 2s\omega_0/Q + \omega_0^2}
\]

where \( \omega_0^2 = 1/(R_1 R_2 C^2) \) and \( Q = \sqrt{(R_2/R_1)} \). Poles are \( p_+ \) and \( p_- \). By taking inverse Laplace transform we get that

\[
v_o(t) = \frac{\omega_0 Q}{p_+ - p_-}(e^{-p_-t} - e^{-p_+t})u(t)
\]

This is just the difference of two decaying exponential when \( Q < 1 \). For the other two cases we have the critically damped case when \( Q = 1 \) and a decaying exponential sinusoid when \( Q > 1 \).

3) For first circuit let \( x = [v \ i]' \). The state equations are

\[
\frac{dv}{dt} = -.01v - i
\]

\[
\frac{di}{dt} = v.
\]

Here we have

\[
A = \begin{bmatrix} -.01 & -1 \\ 1 & 0 \end{bmatrix} \quad (sI - A)^{-1} = \frac{1}{D(s)} \begin{bmatrix} s & -1 \\ 1 & s + .01 \end{bmatrix}
\]

where \( D(s) = \det(sI - A) = s^2 + .01s + 1 \) and the roots of \( D(s) \) are the natural frequencies given by \( s = -.005 \pm j/999975 \).

For second circuit let \( x = [v \ i]' \). Here let node voltage between capacitor and inductor be \( a \). The equations are

\[
\frac{dv}{dt} = -a/2 - i
\]

\[
\frac{di}{dt} = a/4 - i
\]
\[ a = v + \frac{dv}{dt}. \]

Eliminate \( a \) to get state variable equations

\[ \frac{dv}{dt} = -1/3v - 2/3i \]
\[ \frac{di}{dt} = 1/6v - 7/6i \]

Here we have

\[ A = \begin{bmatrix} -1/3 & -2/3 \\ 1/6 & 7/6 \end{bmatrix} \quad (sI - A)^{-1} = \frac{1}{D(s)} \begin{bmatrix} s + 7/6 & -2/3 \\ 1/6 & s + 1/3 \end{bmatrix} \]

where \( D(s) = \text{det}(sI - A) = s^2 + 3/2s + 1/2 \) and the roots of \( D(s) \) are the natural frequencies given by \( s = -0.5, -1. \)

Matlab commands: syms s; phi = inv(s*eye(2)-A); nf = solve(det(s*eye(2)-A))

4) Both circuits are parallel RLC circuits with state equations similar to problem 3a). Here we have \( x = [v \ i]' \). In variable form we have

\[ A = \begin{bmatrix} -1/(RC) & -1/C \\ 1/L & 0 \end{bmatrix} \quad b = \begin{bmatrix} 1/C \\ 0 \end{bmatrix} \quad c = [0 \ 1] \]

a) Here we have \( 1/RC = 1600, \ 1/C = 2 \times 10^5, \) and \( 1/L = 3.2. \)

b) Here we have \( 1/RC = 1600, \ 1/C = 4 \times 10^6, \) and \( 1/L = 0.25. \)

Matlab commands to get overall response: syms s; phi=inv(s*eye(2)-A); xinit= [vinit; iinit]; U = vin/s; Vout= c*phi*b*U + c*phi*xinit; vout=ilaplace(Vout)